

Princess Sumaya University for Technology		جامعة الأميرة سمية للتكنولوجيا
Automatic Control	Student Name:	
Quiz 5	Student ID No.: 2023159	
Date: Summer 29/7/2026	Grade:	

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Q1 The loop transfer function of a single loop feedback system is given by:

$$\frac{128k}{(s+2)^3} = \frac{128k}{s^3 + 6s^2 + 12s + 8}$$

- Plot $L(j\omega)$, $\omega(\infty \rightarrow 0)$ (show details) (3 points)
- Use the Nyquist stability criterion to find the range of positive k for stability (2 points)
- Find the value of k such that the gain margin is 15 dB. (2 points)
- Find the value of k such that the phase margin is 60° (3 points)

$$\lim_{\omega \rightarrow \infty} L(j\omega) = \frac{128k}{(j\omega+2)^3} = 0 \angle -270^\circ, \lim_{\omega \rightarrow 0} \frac{128k}{(j\omega+2)^3} = 16k \angle 0^\circ$$

this is a minimum phase system because all of the poles are on the LHS

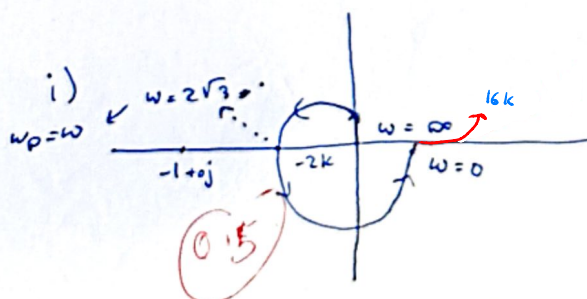
now must find real axis intersection

$$\frac{128k}{-j\omega^3 - 6\omega^2 + 12j\omega + 8} = \frac{128k}{(-6\omega^2 + 8) + j(12\omega - \omega^3)} = \frac{128k [(-6\omega^2 + 8) - j(12\omega - \omega^3)]}{[(-6\omega^2 + 8)^2 + (12\omega - \omega^3)^2]}$$

$$\text{Im} = 0 \Rightarrow -\omega^3 + 12\omega = 0 \Rightarrow \omega_{1,2} = \pm 2\sqrt{3}, \omega_3 = 0 \text{ we only take } \boxed{\omega = 2\sqrt{3}} \quad (1)$$

$$L(j\omega) \Big|_{\omega=2\sqrt{3}} = -2k \text{ so for stability } -2k > -1 \Rightarrow 2k < 1 \quad (2)$$

$$\text{ii) } \boxed{0 < k < \frac{1}{2}}$$



$$\text{iii) } -20 \log |2k| = 15 \text{ dB}$$

$$k = 0.0889 \quad (2)$$

$$\text{iv) } \phi_m = \angle L(j\omega_g) - 180^\circ \Rightarrow \angle L(j\omega_g) = 240^\circ \text{ or } -120^\circ \quad (1)$$

Continuing the solution at the back

$$L(j\omega_z) = \frac{128K}{(j\omega_z + 2)^3}, \text{ we also know that } |L(j\omega_z)| = 1$$

$$\frac{128K}{(\omega_p^2 + 4)^{0.5} \cdot (\omega_p + 4)^{0.5} \cdot (\omega_p + 4)^{0.5}} = 1, \text{ we must find } \omega_p$$

$$-3 \tan^{-1}\left(\frac{\omega_p}{2}\right) = -120 \Rightarrow \frac{\omega_p}{2} = \tan\left(\frac{120}{3}\right)$$

$$\omega_p = 1.67$$

$$\frac{128K}{[(1.67)^2 + 4]^{1.5}} = 1, \text{ solving for } K = 0.138$$